Turbulent impurity transport

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acknowledgements to EFDA JET contributors, Risø group

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- Collisional and neoclassical impurity transport
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  - peaking
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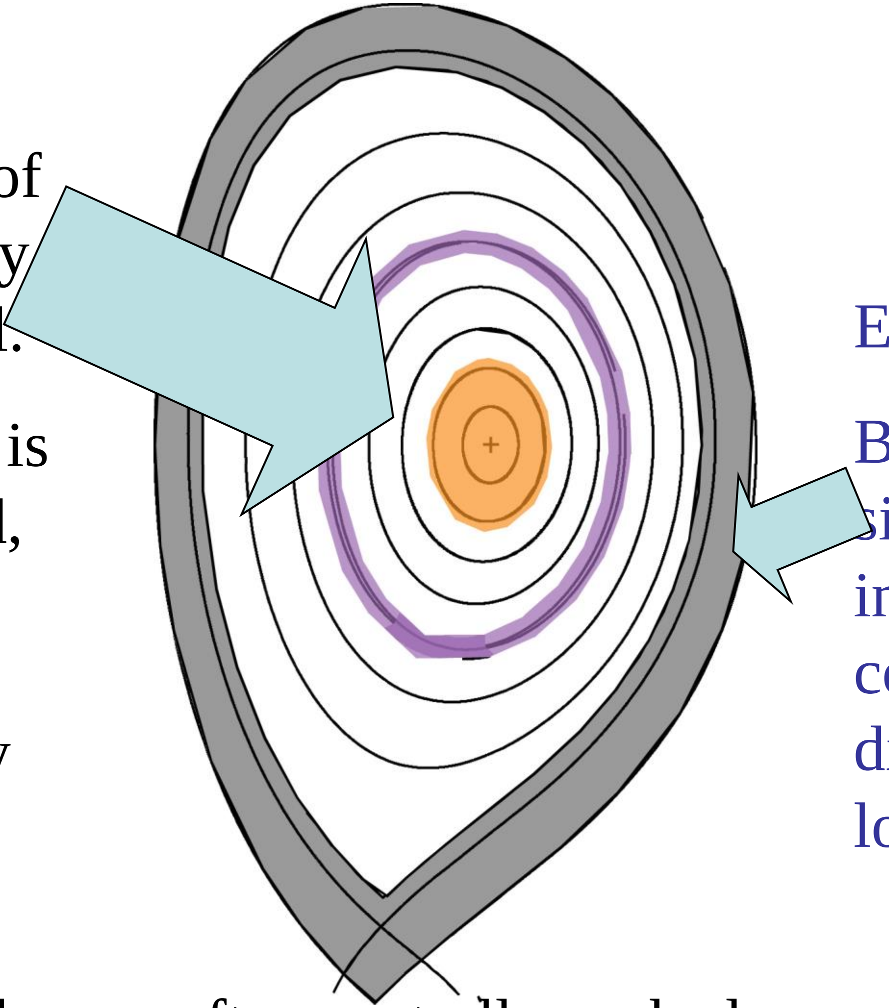
Impurity transport and Fusion

- **impurities are all materials besides the bulk plasma species** (DT)
- impurities originate from the walls
  - intrinsic impurities, source difficult to characterise
- remember:

  bulk plasma material in a typical tokamak corresponds to approx a single atomic layer on the walls
- impurities are sometimes puffed (gases) or injected by ablation (metals).
  - extrinsic impurities, source known, good for experiments to investigate transport.

Different regions in Tokamak



Core: Avoid accumulation of impurities, they dilute and cool.

Accumulation is often observed, especially in plasmas with peaked density profiles

Edge and SOL:

Best to have significant impurities to cool plasma and distribute heat loads

Impurity profiles are often centrally peaked even if the sources are in the edge/SOL region

Impurity transport some remarks (pre turbulent):

- Neoclassical impurity transport theory reviewed in Hirshman and Sigmar (Nuclear Fusion 21, 1079, 1981)
- includes all transport effects arising to the confinement geometry of the tokamak.
  - this includes plasma flows
  - ”the radial electric field determines via equilibrium, the toroidal toroidal velocities of all species” (Houlberg et al. PoP 1997)
- Impurity transport is complicated by the fact, that
  - Impurities come in many charge states
  - transition probabilities are not always known (f.x. for higly ionised states of W, the ITER divertor material)

Results of neoclassical theory

- Impurities will be highly peaked if density profile is peaked.
- peaking is proportional to Z

Does turbulence save us?

Principle to characterise transport:

- Linear relationship assumed between impurity flux and density gradient

$$\Gamma_z = -D_z \frac{\partial n_z}{\partial r} + n_z V_z$$

Convection coefficient

Vz >0 outwards

Diffusion coefficient

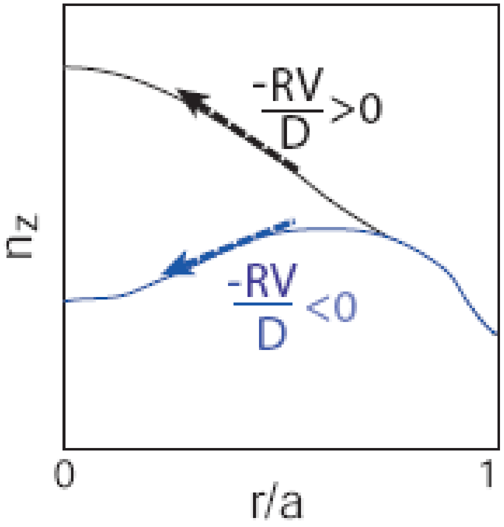
- In steady-state conditions and with edge source the local impurity density gradient length:

$$\Gamma = 0 \Rightarrow \frac{-\partial n_z}{n_z \partial r} = \frac{-V_z}{D_z}$$

Peaking factor

$$\frac{-RV_z}{D_z}$$

R major radius device



Beware of effective diffusion coefficients!

Peaking is not all: Source level is important as well!

Classical Particle Dispersion

Single particle dispersion:

G.I. Taylor 1915

Stationary, homogeneous turbulent flows:

→

$$R^2(t) \equiv \langle (\vec{r}(t) - \vec{r}(t_0))^2 \rangle$$

$$R^2(t) = 2\langle v^2 \rangle \int_0^t (t - \tau) C_L(\tau) d\tau$$

Lagrangian integral time scale; $\tau_L = \int_0^\infty C_L(\tau) d\tau$

$$C_L(t) = \langle \vec{v}(\tau + t) \cdot \vec{v}(\tau) \rangle / \langle \vec{v}^2(\tau) \rangle$$

Two limits:

→

$$t \ll \tau_L \ : \ R^2(t) = \langle v^2 \rangle t^2$$

$$t \gg \tau_L \ : \ R^2(t) = 2Dt$$

Diffusion coefficient: $D = \langle v^2 \rangle \tau_L$

General: $R^2(t) \propto t^\alpha$

$\alpha > 1$ **Superdiffusion** $\alpha < 1$ **Subdiffusion**

Fick's law:

→

$D_{eff} = \Gamma_0 / \nabla n_0$, with normalizations $D_{eff} = \Gamma_0$.

Impurities as passive tracers

- Simplemost problem: One species, density small compared to bulk
- Impurity density is low
- it does not contribute to charge neutrality
- Electromagnetic fields in plasma are not influenced by the impurities.

Passive scalar problem

(Reviews in fluid dynamics by Falkovich and Warhaft)

Does passive tracer transport represent bulk species transport (consistency)?

Is dispersion equal to transport?

Particle dispersion in 2D drift wave turbulence

Hasegawa-Wakatani equations (HWE): the resistive drift wave instability (PRL **50**, 682 (1983)):

$$\partial_t n + \partial_y \varphi + \{\varphi, n\} = -C\,(n - \varphi) + \mu_n \nabla^2 n$$

$$\partial_t \nabla^2 \varphi + \{\varphi, \nabla^2 \varphi\} = -C\,(n - \varphi) + \mu_\varphi \nabla^4 \varphi$$

$1/C = 1/k_\|^2 L_\|^2 \; L_\| = (L_n T_e / m_e c_s \nu_{ei})^{1/2}$

$\{\varphi, \psi\} \equiv \hat{z} \times \nabla\varphi \cdot \nabla\psi = \frac{\partial\varphi}{\partial x}\frac{\partial\psi}{\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial\varphi}{\partial y}$

$u = -\frac{\partial\varphi}{\partial y};\; v = \frac{\partial\varphi}{\partial x}$

Normalization: $\rho_s = c_s/\Omega_i$ for lengths; L_n/c_s for the times;

$c_s = \sqrt{T_e/m_i};\; L_n = |(\nabla n_0(x)/n_0(x))^{-1}|$

$(T_e/e)\,(\rho_s/L_n)$ for potential $n_0\rho_s/L_n$ for density; $\mu_n = \mu_\varphi = \mu$.

Trace particles vs. impurity field

Up to 100.000 particles are adverted in resistive drift-wave turbulence – Hasegawa-Wakatani (PRL 1983) model 2D.

$$\vec{x}(t) = \vec{x}_0 + \int_0^t \vec{v}(\vec{x}(t'), t')dt$$

Principal component of $\vec{v} = (u, v)$ is the $E \times B$-velocity, $\vec{v}_E$:: Ideal inertia-less particles.

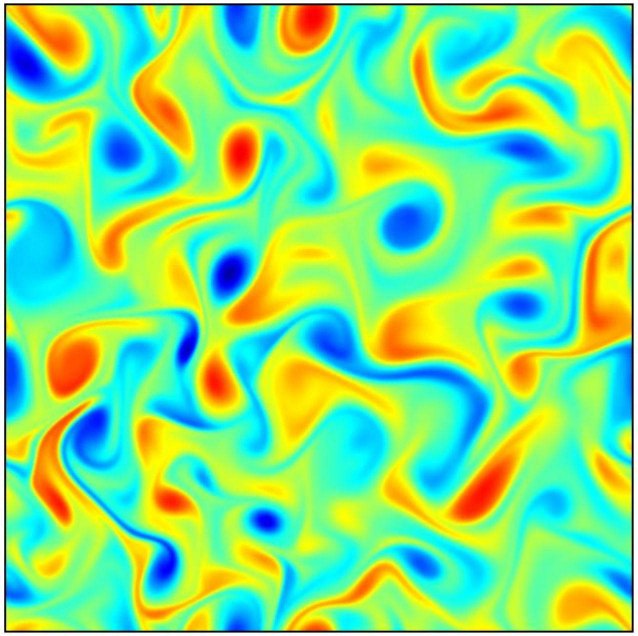
Inertial effects : adding the polarization drift, $\vec{v}_p$

$$\vec{v}_p = -\zeta \left(\frac{\partial}{\partial t} + (\vec{v}_E \cdot \nabla) \right) \nabla \varphi$$

$$\zeta = \frac{eM}{qm_i} \frac{\rho_s}{L_n}$$

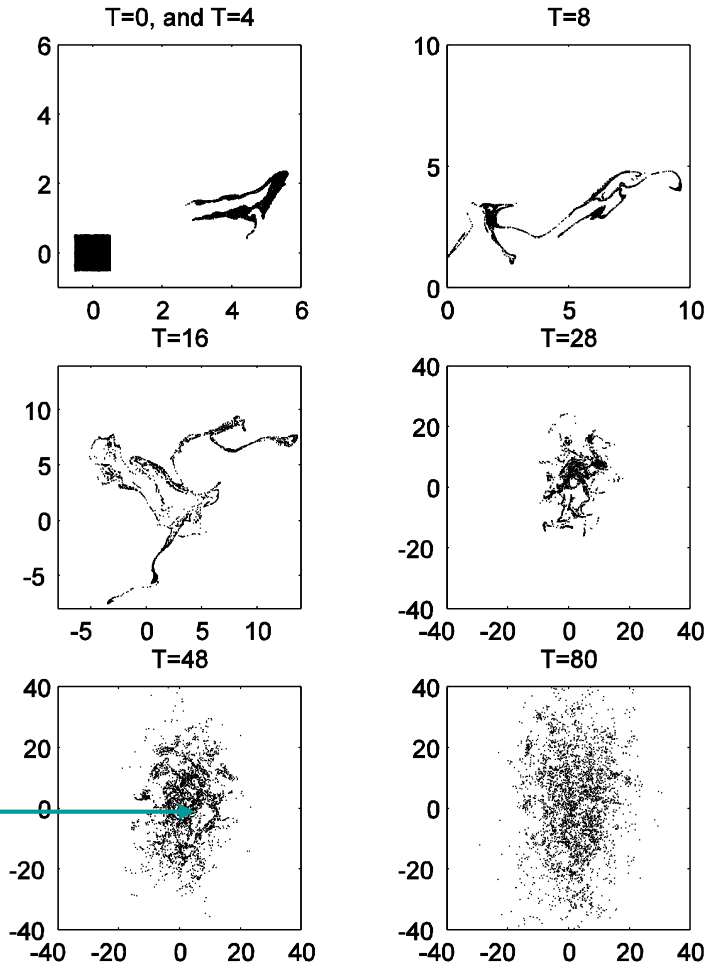
Important for heavier impurities!

Particle dispersion in plasma turbulence



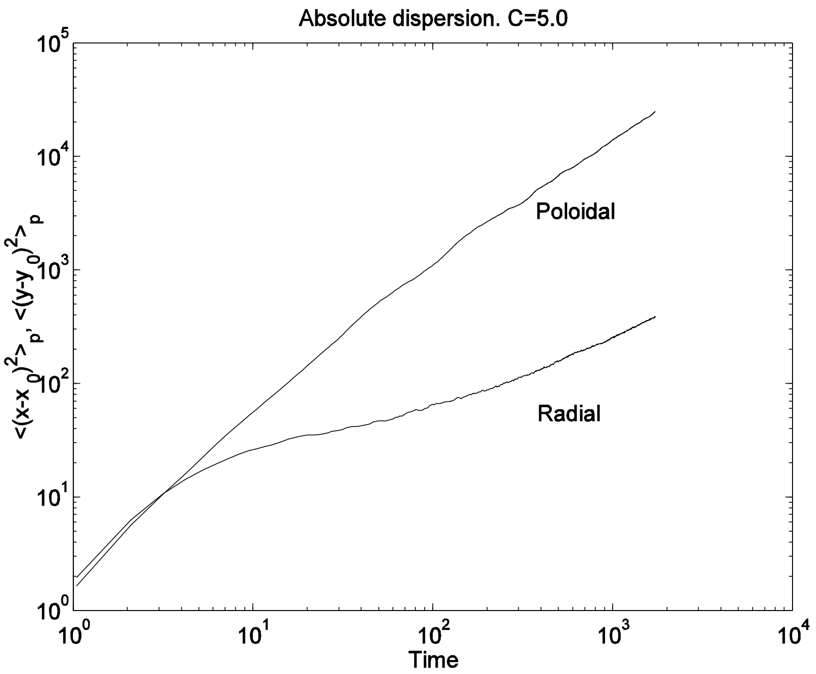
Vorticity

Drift-wave turbulence

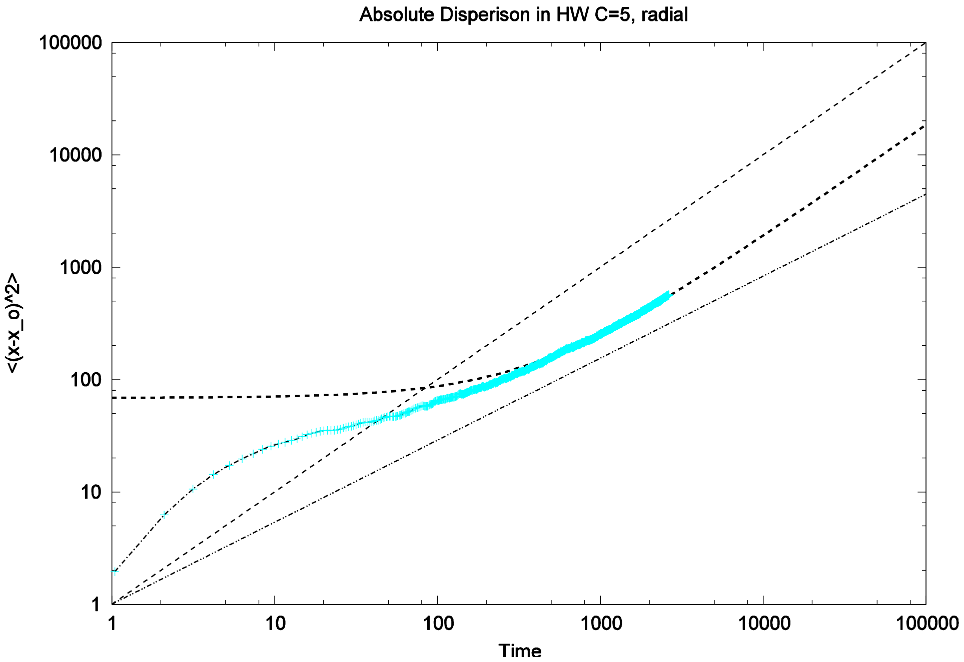


Basu et al Phys. Plasmas 10, 2696 (2003)

Particle dispersion



The mean square particle displacement radially and poloidally.



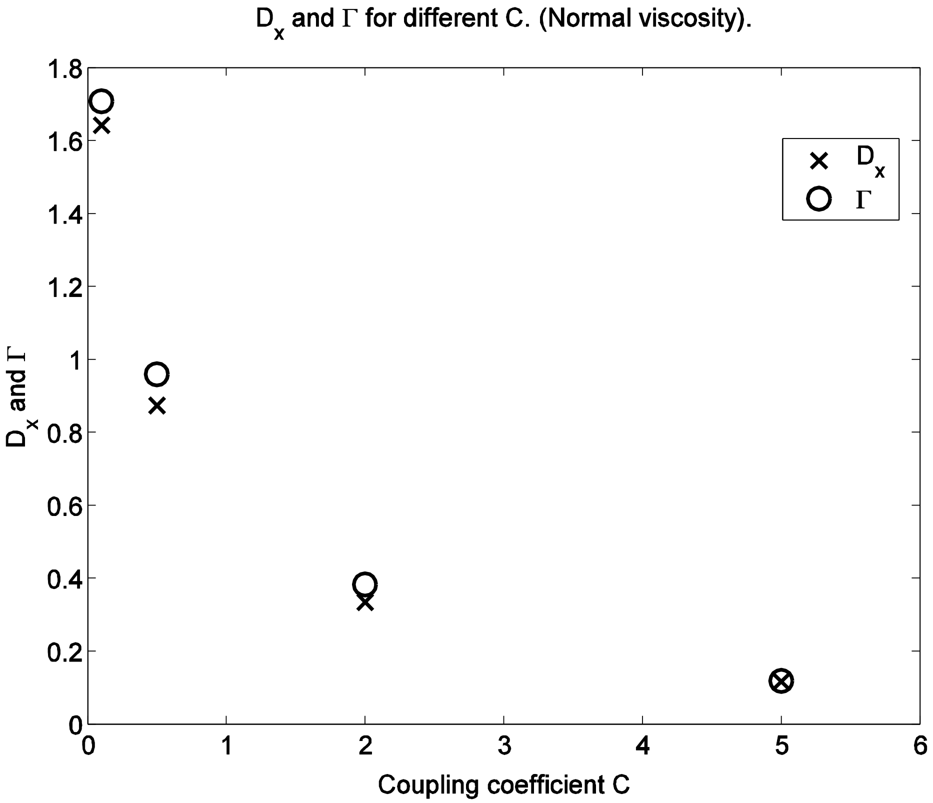
The radial particle displacement.

Fit: $At\beta + B$ for $t > 400$; $\beta = 1$.

Asymptotically normal diffusion!

Naulin, Nielsen et al, PoP 2002

Tracer diffusion coefficient and particle flux



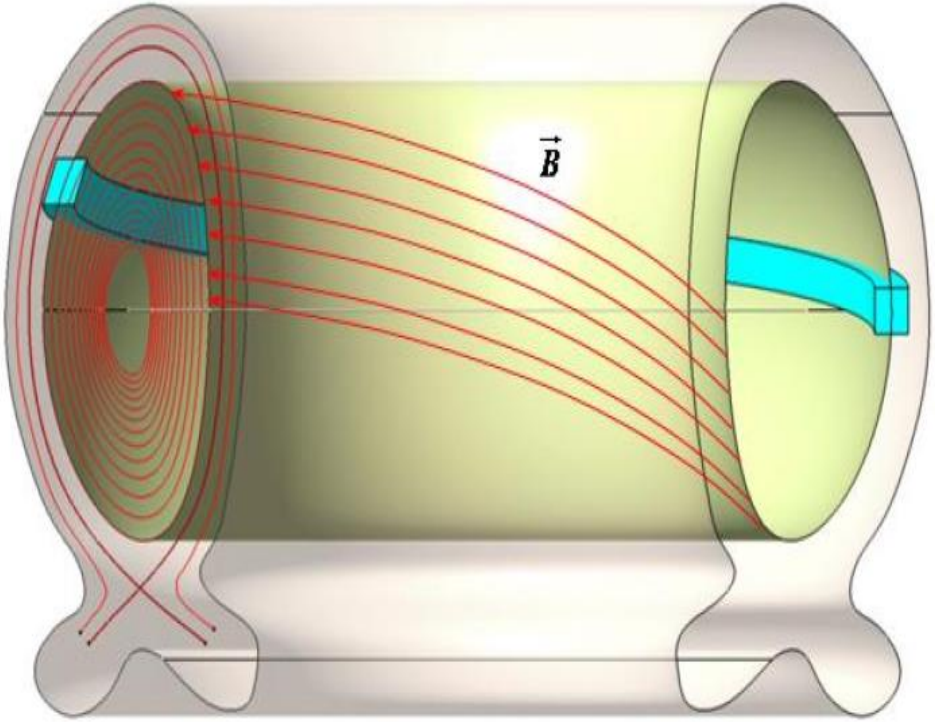
Comparison between D for tracer particles and D from the flux Γ; D = $\Gamma/d_x n_0$

Consistency check for passive tracer simulations!

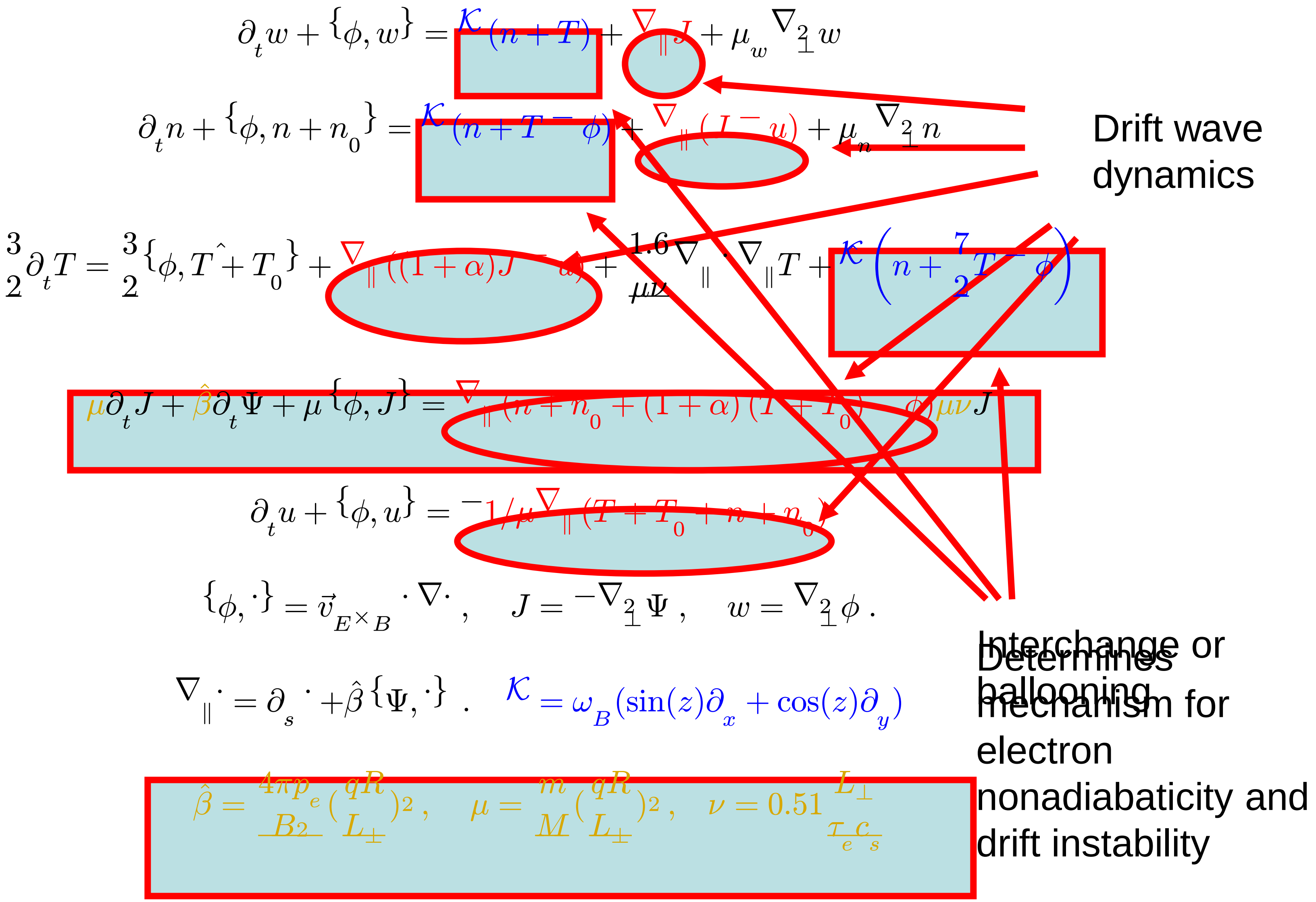
Tracer transport = bulk transport for same Z

In toroidal Geometry

- Flux tube: x (radial) and y (toroidal) determine drift plane, parallel coordinate follows field line poloidally around the torus
- local magnetic shear
- B field decays outwards, curvature



3D equations (bit complex, sorry…)



$$\partial_t w + \{\phi, w\} = \mathcal{K}(n+T) + \nabla_\parallel J + \mu_w \nabla_\perp^2 w$$

$$\partial_t n + \{\phi, n + n_0\} = \mathcal{K}(n+T-\phi) + \nabla_\parallel (J-u) + \mu_n \nabla_\perp^2 n$$

$$\frac{3}{2}\partial_t T = \frac{3}{2}\{\phi, T + T_0\} + \nabla_\parallel ((1+\alpha)J - u) + \frac{1.6}{\mu\nu}\nabla_\parallel \cdot \nabla_\parallel T + \mathcal{K}\left(n + \frac{7}{2}T - \phi\right)$$

$$\mu \partial_t J + \hat{\beta}\partial_t \Psi + \mu\{\phi, J\} = \nabla_\parallel (n + n_0 + (1+\alpha)(T + T_0) - \phi) - \mu\nu J$$

$$\partial_t u + \{\phi, u\} = -1/\mu \nabla_\parallel (T + T_0 + n + n_0)$$

$$\{\phi, \cdot\} = \vec{v}_{E\times B} \cdot \nabla\cdot \,, \quad J = -\nabla_\perp^2 \Psi \,, \quad w = \nabla_\perp^2 \phi \,.$$

$$\nabla_\parallel \cdot = \partial_s \cdot + \hat{\beta}\{\Psi, \cdot\} \,. \quad \mathcal{K} = \omega_B (\sin(z)\partial_x + \cos(z)\partial_y)$$

$$\hat{\beta} = \frac{4\pi p_e}{B^2}\left(\frac{qR}{L_\perp}\right)^2 , \quad \mu = \frac{m}{M}\left(\frac{qR}{L_\perp}\right)^2 , \quad \nu = 0.51 \frac{L_\perp}{\tau_e c_s}$$

Drift wave dynamics

Interchange or ballooning

Determines mechanism for electron nonadiabaticity and drift instability

Impurity equation

$$d_t n_{imp} = M\nabla_\perp \cdot (n_{imp} d_t \nabla_\perp \phi) - n_{imp}\mathcal{K}(\phi) - \nabla_{||}(n_{imp} u)$$

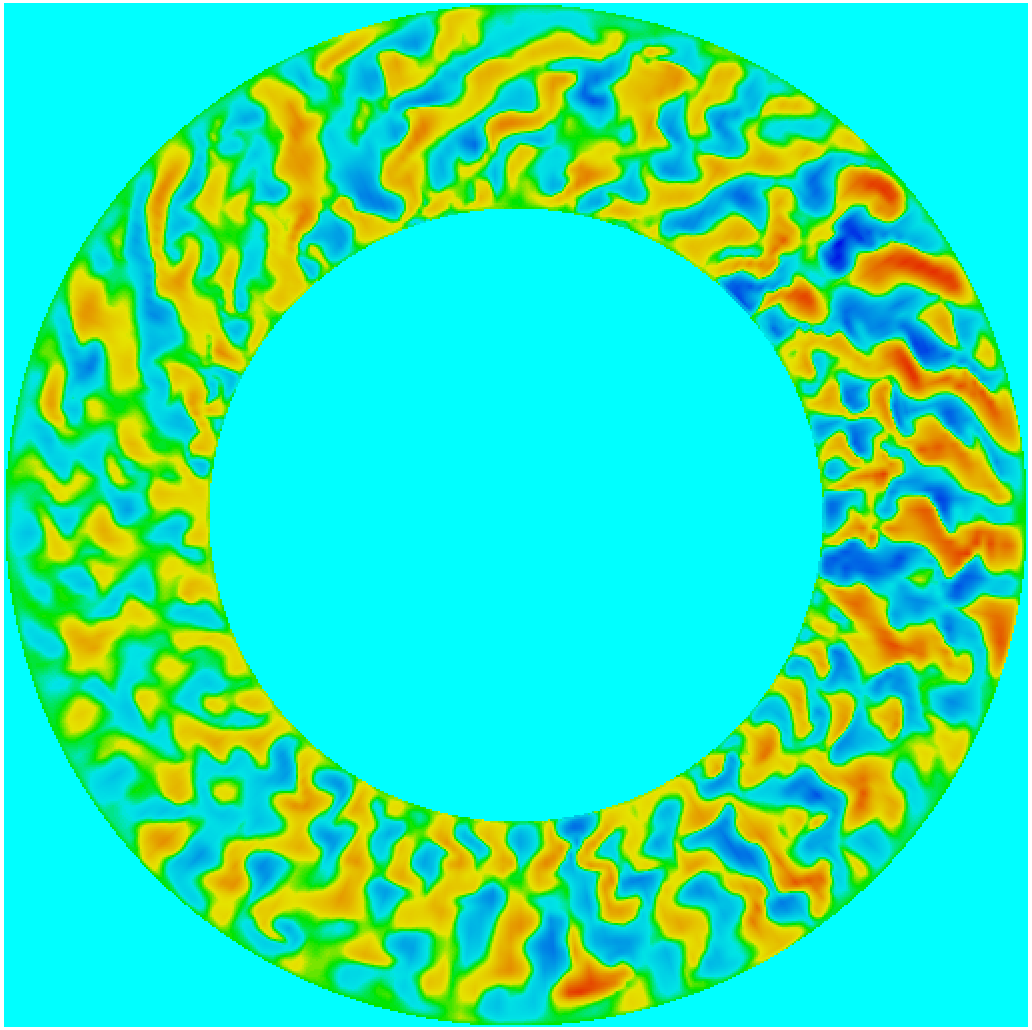
d_t includes full $E \times B$ advection.
Polarisation drift term includes relative mass and charge state of impurity

$$M = M_{imp}/(Zm_{ion})$$

Note:
parallel convection included cold impurities
polarisation drift dropped for reasons of resolution....
Initial condition: Poloidally and toroidally constant Gaussian pulse

$$n_{imp}(t=0) \approx \exp(-r^2/\sigma_0)$$

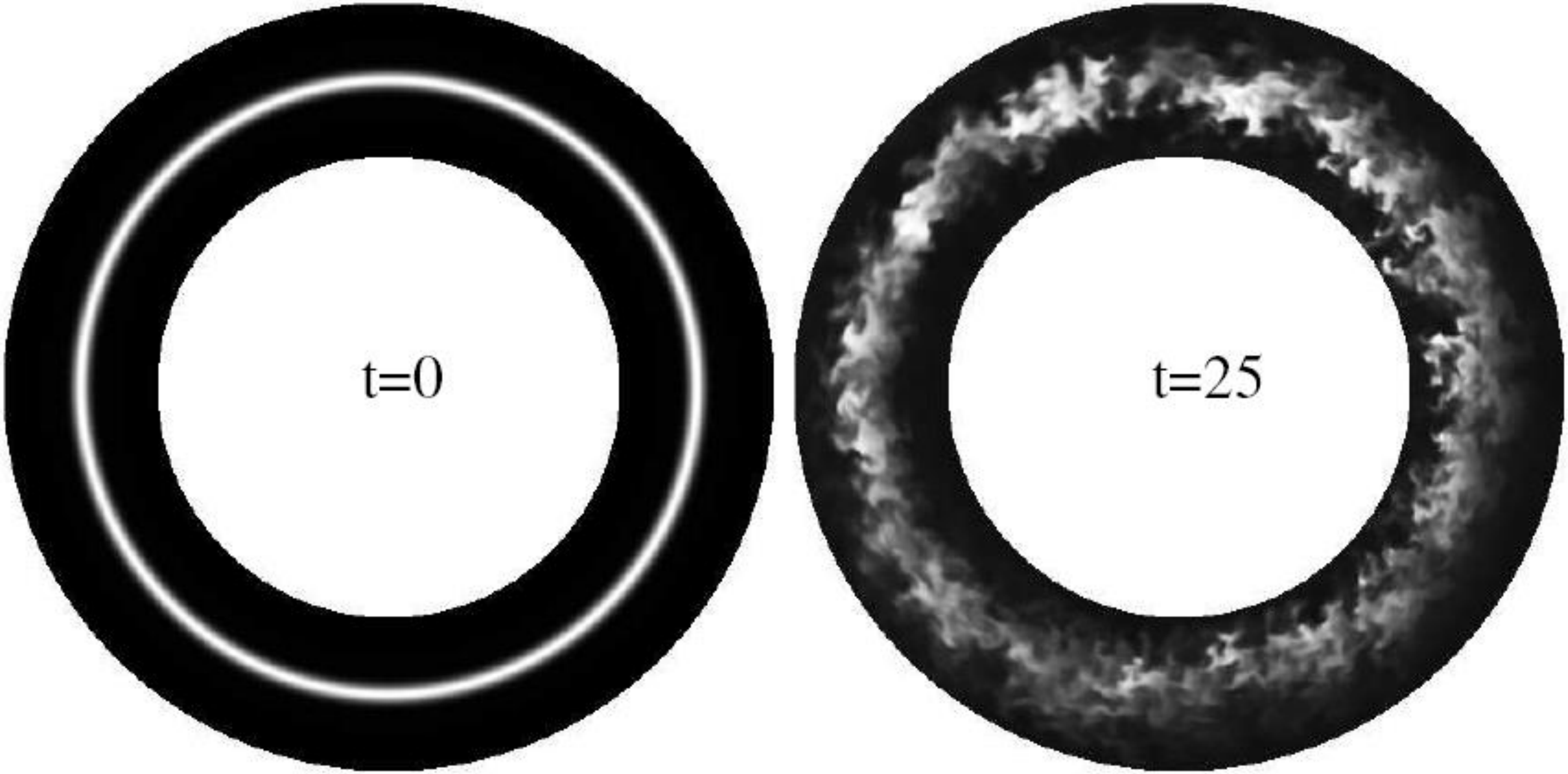
Poloidal visualisation of plasma density fluctuations



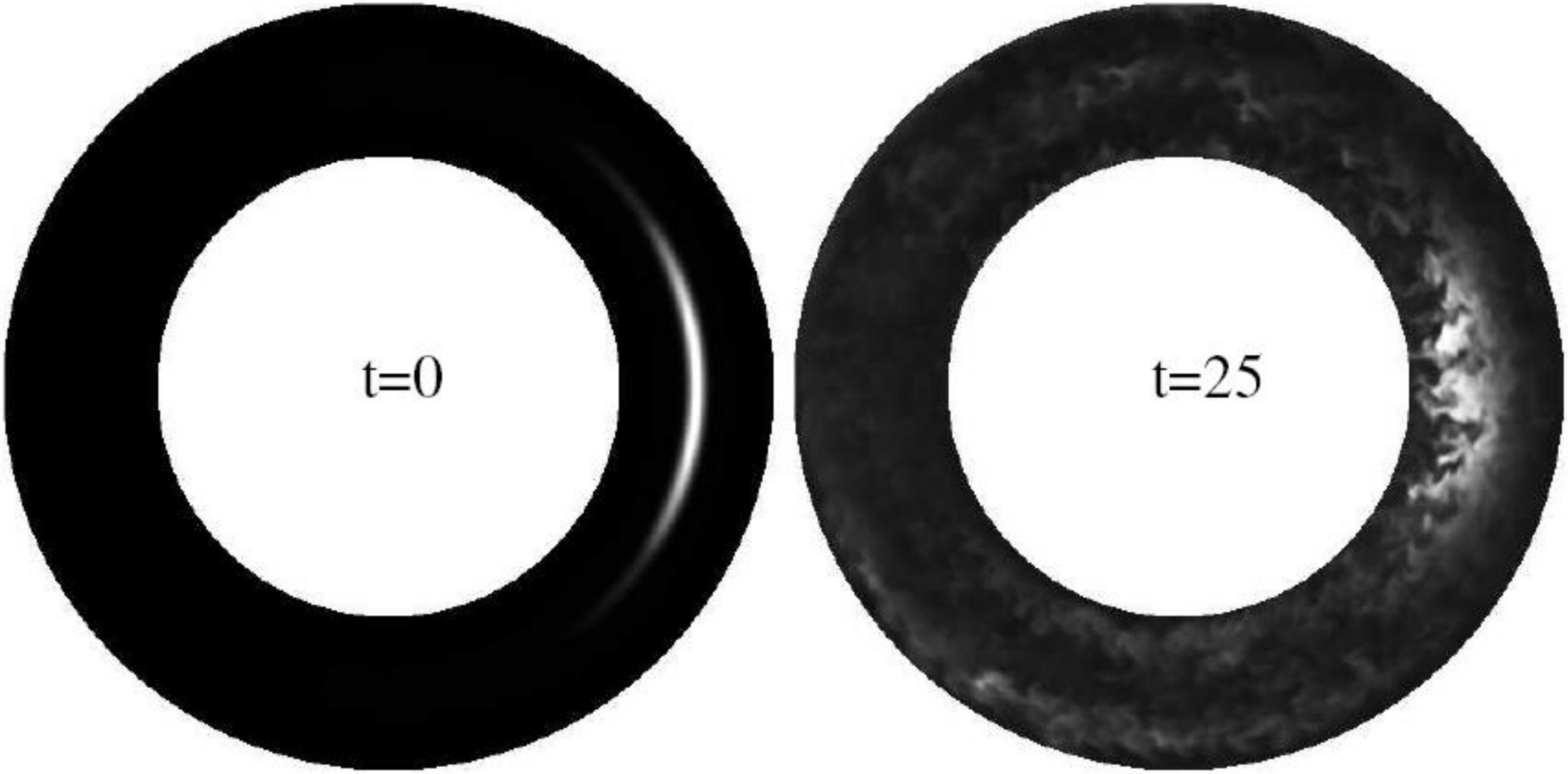
Density fluctuations in a poloidal projection

Projection shows that this is enough to resolve poloidal variation of geometry, ballooning and some magnetic shear effects visible.

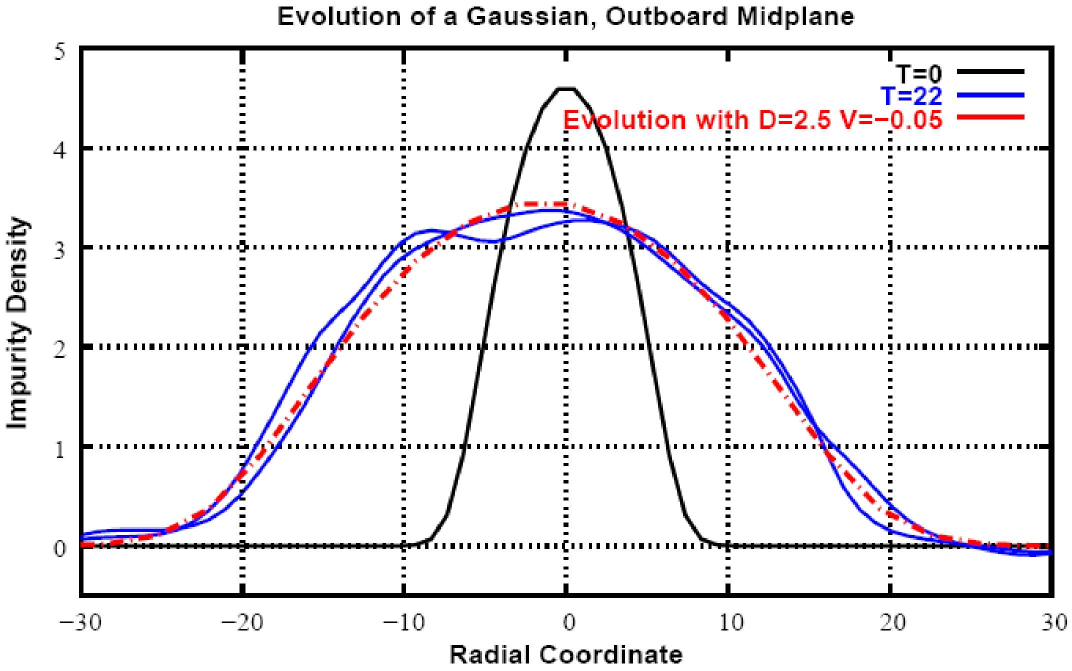
Development of impurity density



Impurity density, localised poloidally



Curvature Pinch



Evolution of impurity profile reveals diffusion (spreading) and pinch (central peak motion). Values for D and V can be fitted.

Naulin et al., PRE 2005

Turbulent Equipartition

absence of parallel convection, finite mass effects and diffusion

approximate Lagrangian invariant

$$L(s) = \ln n_{imp} + \omega_B cos(s)x - \omega_B sin(s)y\ .$$

Turbulent EquiPartition

assumes the spatial homogenization of L by the turbulence.

$$< \ln n_{imp} >_y = -\omega_B cos(s)x$$

This is known as the Curvature Pinch.

For systems with trapped particles the proportionality is with q.

Frojdh NF 1992, Isichenkov PRL 1996, Naulin 1998

Thermodiffusion

$$d_t n_Z = -\omega_B \left(n_Z \nabla \phi + \frac{1}{Z} \nabla p_Z \right)$$

$$d_t p_Z = -5/3\,\omega_B \left(p_Z \nabla \phi + \frac{1}{Z} \nabla p_Z^2 / n_Z \right)$$

- use impurity temperature gradient in addition to density equation (Nordmann, Weiland, NF, 1989)
- evaluate quasilinear fluxes.
- determine impurity density response to external potential (made by some instability)

Quasilinear fluxes (Transport matrix also gives pinches)

- Practical expressions of fluxes

$$\begin{pmatrix} \dfrac{\Gamma_i}{n_i} \\ \dfrac{Q_i}{n_i T_i} \end{pmatrix} = -\begin{bmatrix} D_{nn} & D_{nT} \\ D_{nT} & D_{TT} \end{bmatrix} \cdot \begin{pmatrix} \dfrac{\partial_r n_i}{n_i} + \dfrac{2}{R} \\ \dfrac{\partial_r T_i}{T_i} + \dfrac{2(\Gamma - 1)}{R} \end{pmatrix}$$

$$D_{nn} \approx D_{turb} \quad ; \quad D_{TT} \approx \frac{1}{\Gamma - 1} D_{turb}$$

$$D_{nT} \approx \left\langle \frac{\omega}{\omega_{di}} - \Gamma \right\rangle = \sum_{\mathbf{k}} \frac{\left|\tilde{v}_{E\mathbf{k}}\right|^2}{\Delta\omega_{\mathbf{k}}} \frac{2(\omega_{\mathbf{k}} - \Gamma\omega_{di})\omega_{di}}{\Delta\omega_{\mathbf{k}}^2}$$

- D_{nT} sign depends on fluctuation spectrum.

Dubuit, Garbet, TTF 2006, PoP 2007

Quasilinear result:

$$\Gamma_{Z\mathrm{turb}} = -D_Z\left\{ \underset{\substack{\uparrow\\ \text{Diffusion}}}{\frac{dn_Z}{dr}} + \underset{\substack{\uparrow\\ \text{Curvature pinch}}}{C_q(s)\frac{2}{R}n_Z} - \underset{\substack{\uparrow\\ \text{Thermodiffusion}}}{\frac{C_T(\omega)}{Z}\frac{dT_Z}{T_Z dr}n_Z} \right\}$$

Additional term appears if parallel convection is added (Angioni and Peeters, PRL 2006)

$$\frac{VR}{D} = -\left[2\left|\frac{\omega}{\omega_d}\right|^2 + 4\frac{\omega_r}{\omega_d}\frac{Z}{A}\left(\frac{k_{//}v_{TD}}{\omega}\right)^2 \right]$$

Thermodiffusion and parallel pinch depend in sign on frequency of bulk turbulence.

Impurity density with finite inertia

The influence of inertia enters via the polarization.

$$(\partial_t + \mathbf{v}_E \cdot \nabla) n_{imp} = \zeta \nabla \cdot (n_{imp} (\partial_t + \mathbf{v}_E \cdot \nabla) \nabla \phi) + \mu \nabla^2 n_{imp}.$$

Restriction $n_{imp} \ll n$!

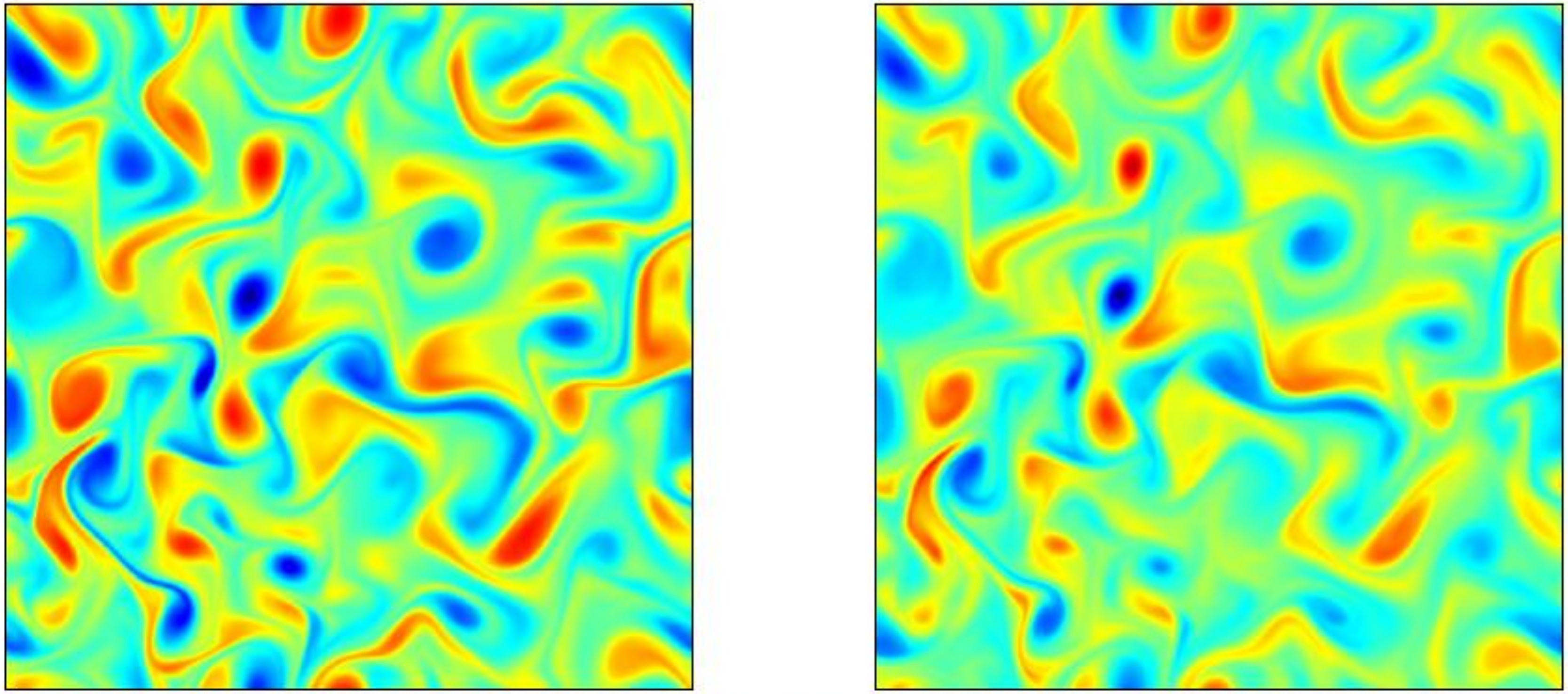
Lagrangian invariant: $(\partial_t + \mathbf{v}_E \cdot \nabla)(\ln n_{imp} - \zeta \omega) \approx 0$

Turbulent mixing will homogenize the Lagrangian invariant :

$\ln n_{imp} - \zeta \omega \approx const.$

Inertial impurities cluster in vortices

2D-drift wave turbulence: Aggregation/expulsion of particles in/from vortices

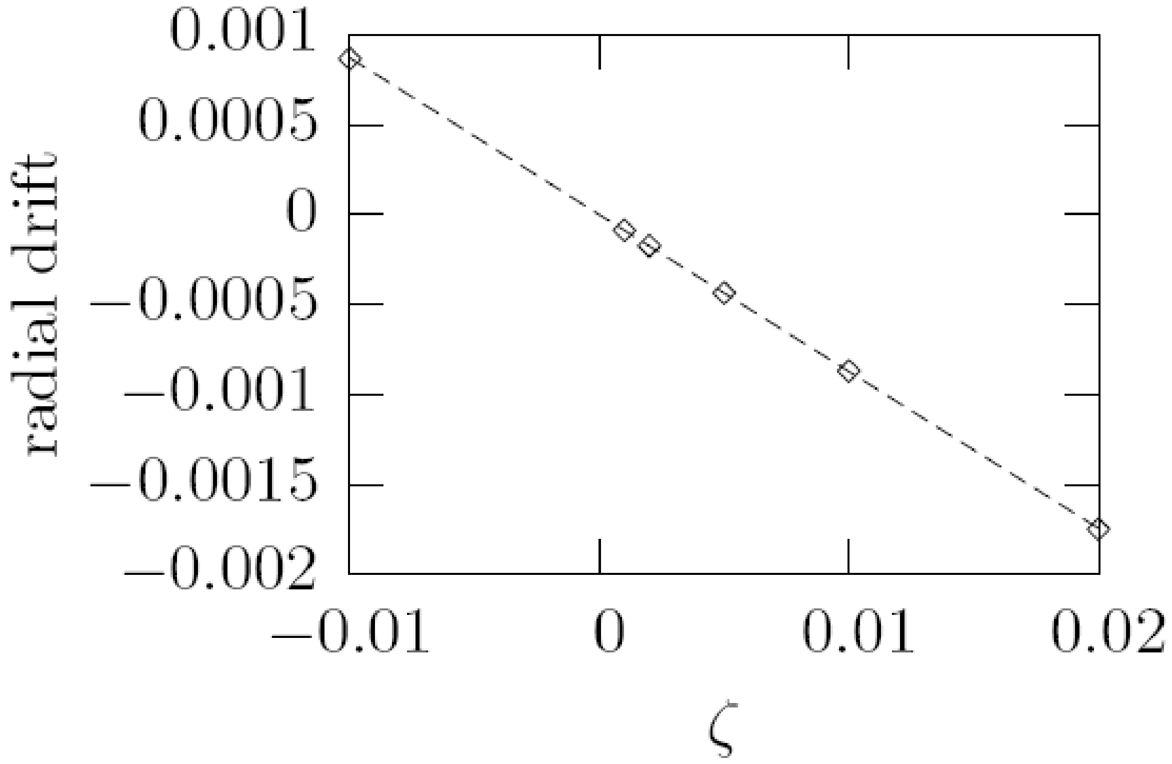


Vorticity and density of inertial impurities in the saturated state with $C = 1$ and $M = 0.01$, $L = 40$, $\mu_n = \mu_\varphi = \mu_\theta = 0.02$.

Priego et al. PoP 2005

Inertia related weak up-gradient pinch

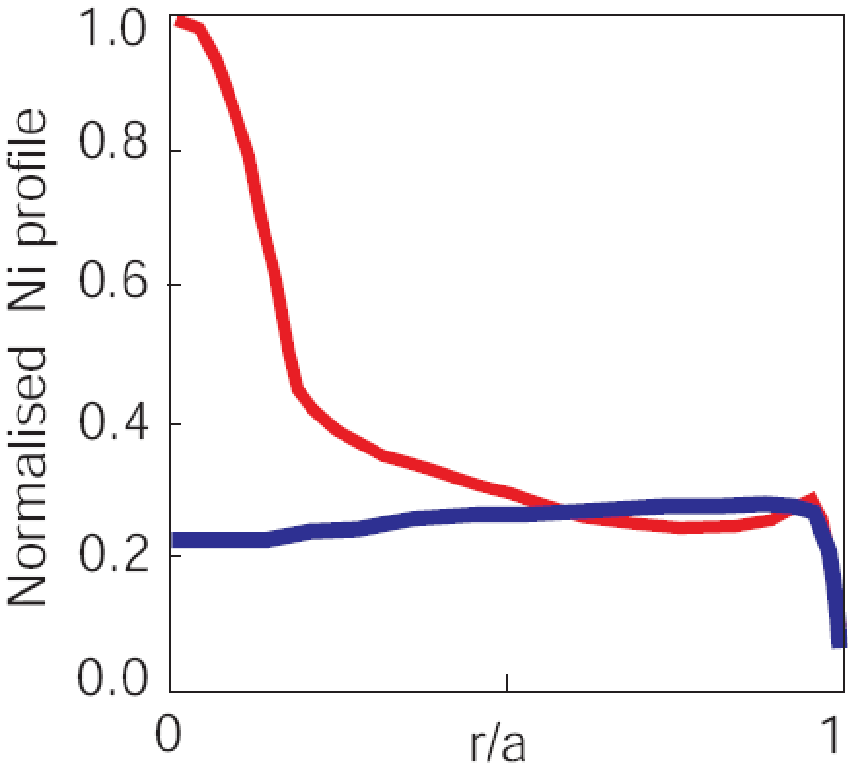
Considering the collective drift of impurities.



The radial drift velocity has a definite sign that depends on the sign of ζ. Anomalous pinch which shows linear effective mass scaling.

Priego, PoP 2005

Experimental results



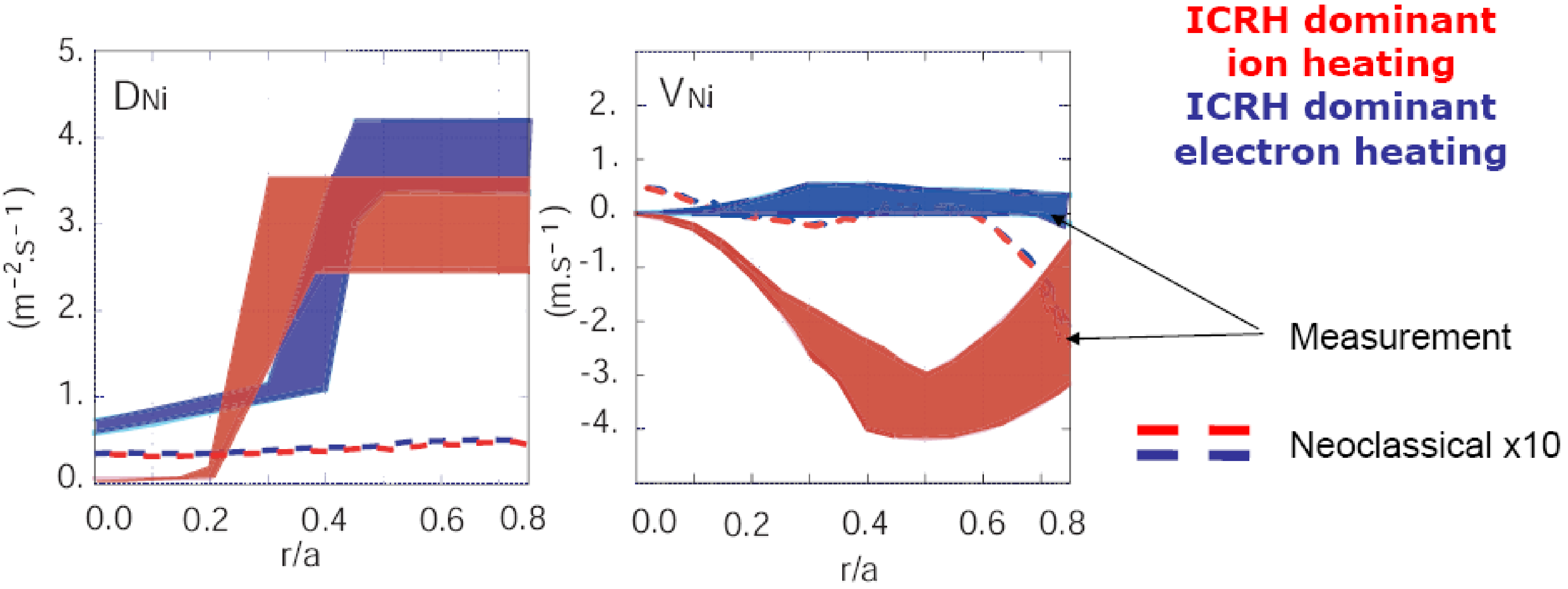
Steady-state profile calculated from D and V

ICRH dominant ion heating
Peaked Ni profile

ICRH dominant electron
Slightly hollow Ni profile

[M-E. Puiatti PoP 13 2006]

Interpretation of experimantal results



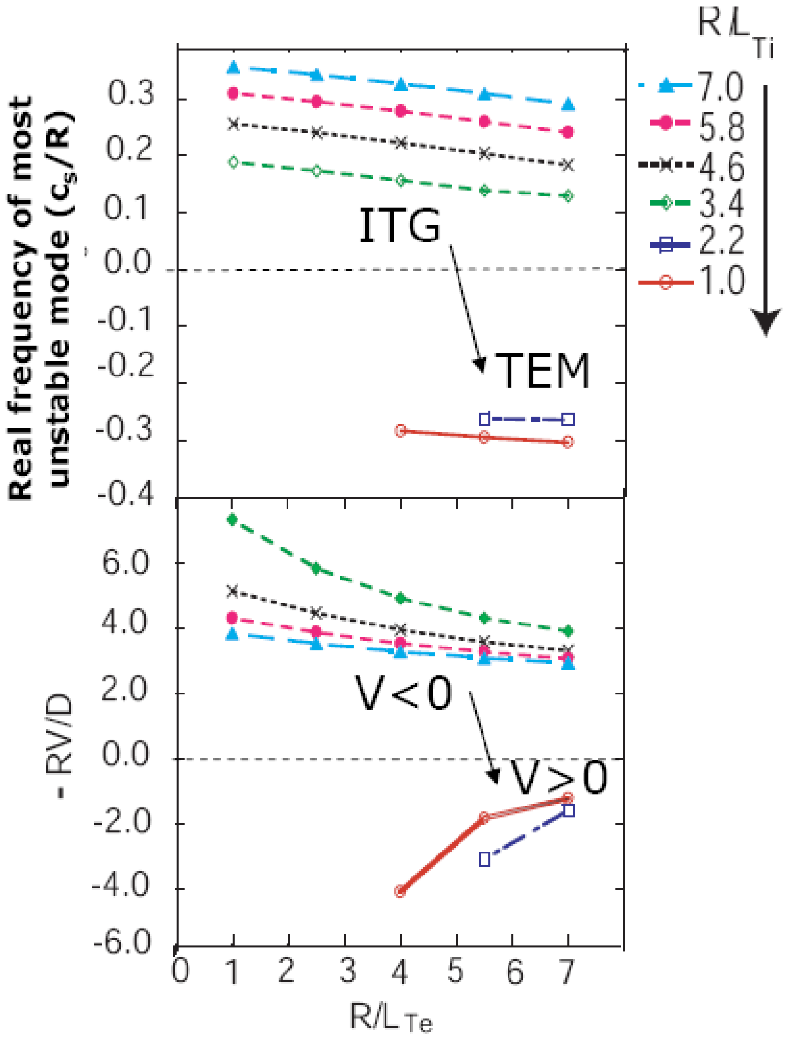
- Diffusion increased in centre
- Convection reversed at mid-radius

While neoclassical transport unchanged

→ Reduction in Ni peaking due to anomalous transport

[M-E. Puiatti PoP 13 2006]

Pinch reversal in Gyrokinetic code (GS2)



- **Investigate** transition from ITG to R/LTe driven TEM
  - Stabilised R/Ln driven TEM: R/Ln=2.
  - gradually decreasing R/L_{Ti} towards stabilisation of ITG modes.

Te/Ti=0.95, R/Ln=2

- Reproduce a pinch reversal as observed experimentally

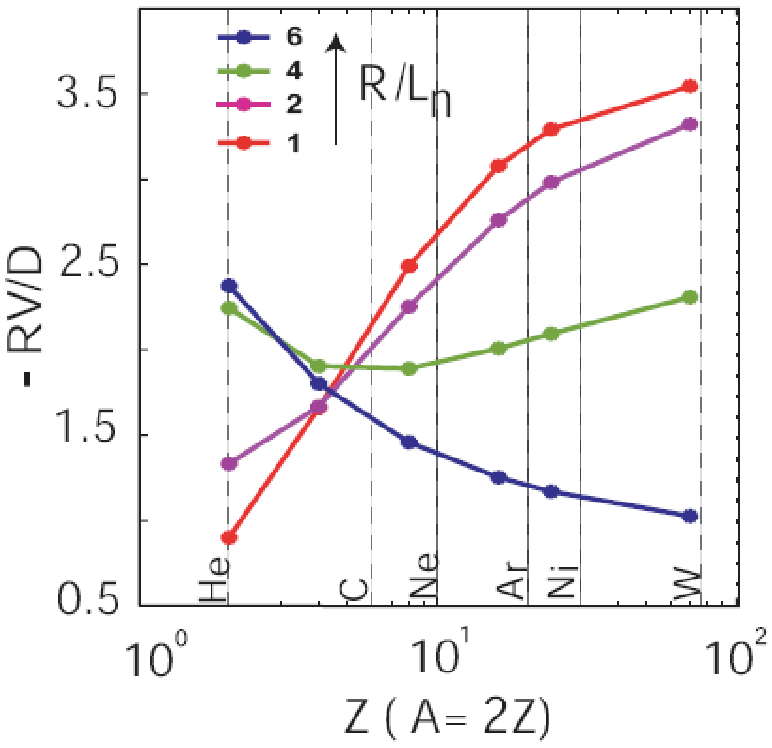
[C. Angioni PRL. 96 2006]
[M-E. Puiatti PoP 13 2006]

in Gyro [Estrada-Mila, Candy, Waltz PoP 2005]

Z dependence (first steps with gyrokinetic code)

GS2

[R/LTi=7, R/LTe=6, Te/Ti=0.88]



[C. Angioni]

- D and V calculated with the linear version of the gyrokinetic code GS2:
  - trace impurity considered.
  - only the fastest growing mode is taken in the quasi–linear model
  - no neoclassical transport included.

- Complex trend in Z of turbulent transport

➔ specific calculation needed for studied discharge

Summary: developments in anomalous impurity transport

- Three main mechanisms have been identified

Curvature pinch[1]	Compressibility of ExB drift velocity	Independent on Z and A
Thermodiffusion pinch[2]	Compression of the diamagnetic drift velocity $\alpha \frac{\nabla Tz}{Tz}$	Dependent of 1/Z
Pinch connected to the parallel dynamics of the impurity[3]	Compression of parallel velocity fluctuations produced along the field line by the fluctuating electrostatic potential	Dependent on Z/A

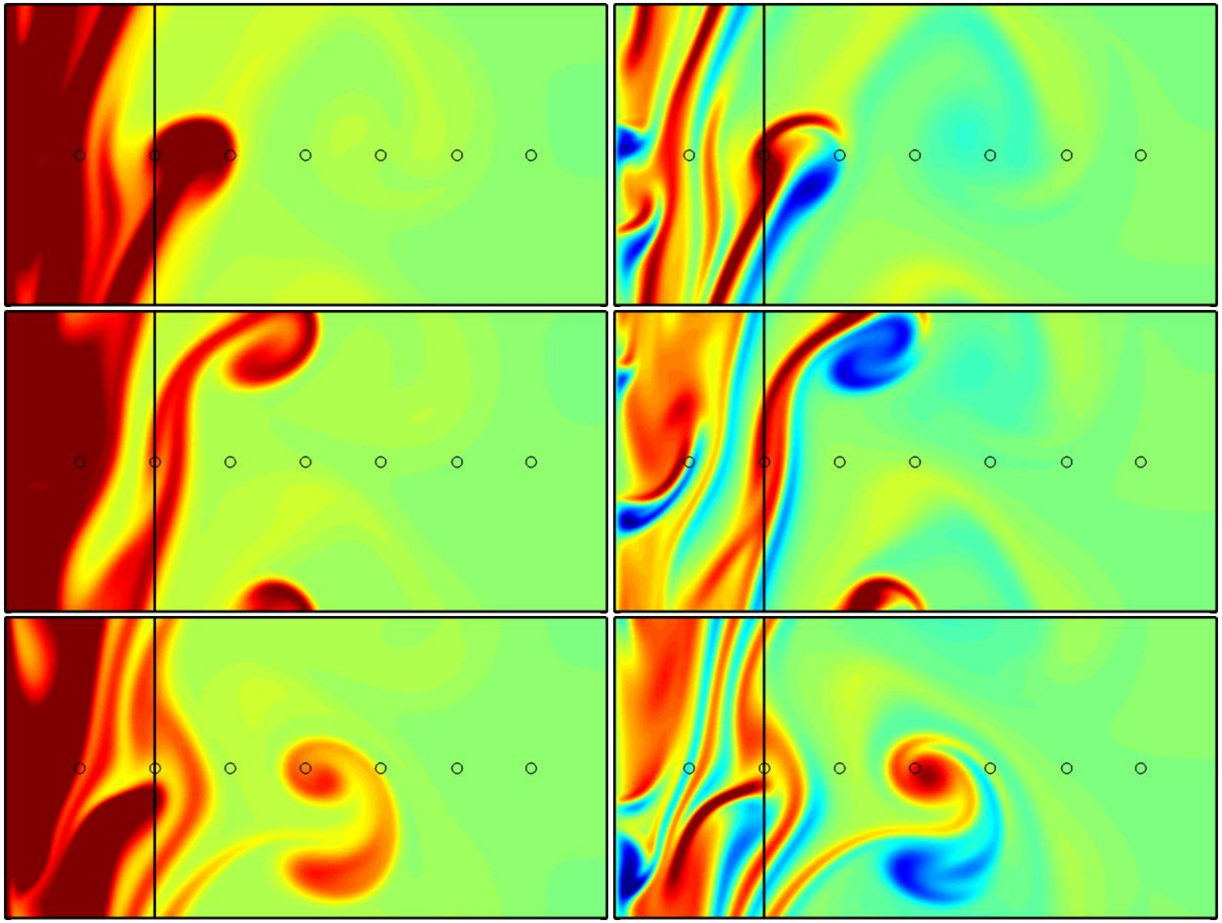
[1] [J. Weiland NF 29 1989]
[1] [X. Garbet PRL 91 2003]
[1] [M. B. Isichenko PRL 1996]
[1] [D.R. Baker PoP 5 1998]
[1] [V. Naulin Phys Rev. E 2005]
[2] [M. Frojdh NF 32 1992]
[2] [X. Garbet PoP 12 2005]
[2,3] [C. Angioni C PRL. 96 2006]

Giroud, IAEA 2006

Conclusions

- Impurity transport important to clarify for ITER
- Passive tracer dynamics captures transport consistently
- Pinches result from:
  - velocity compressibilities (curvature, parallel v)
  - thermodiffusion (temperature gradient flattening)
- Quasilinear pinches depend on direction of wave turbulence
- Impurity control, power levels?

Spatial structure during a burst



Particle density (left) and vorticity (right) during a burst (Δt = 500)

Dynamics of tracer particles

The passive tracer particles may model impurity dynamics, in the limit of no back-reaction on the plasma dynamics:

Impurity density is much lower than the plasma particle density.

(Naulin PRE ’05; Priego *et al* PoP ’05; Naulin *et al* Physica Scripta ’06)

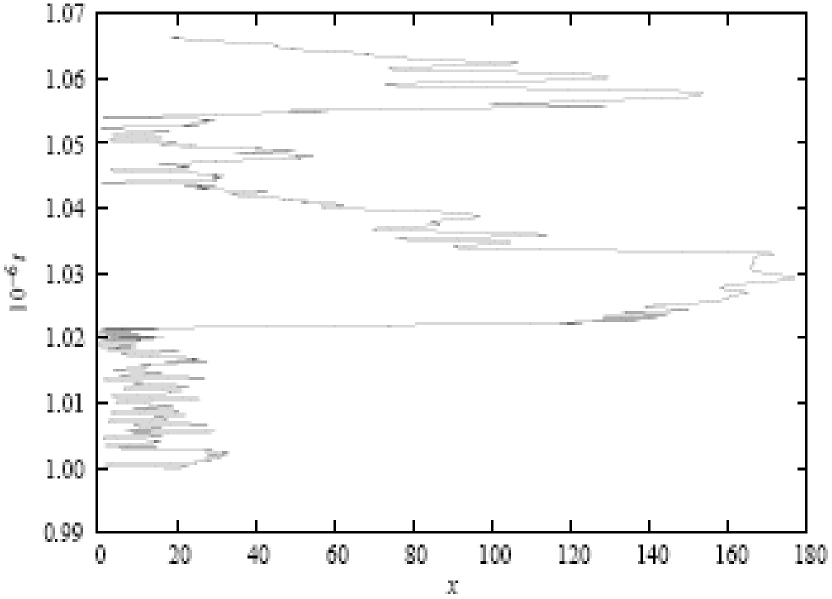
Particles are advected as:

$$\frac{d\vec{x}}{dt} = \vec{v} = \frac{1}{B(x)}\hat{z} \times \nabla\phi$$

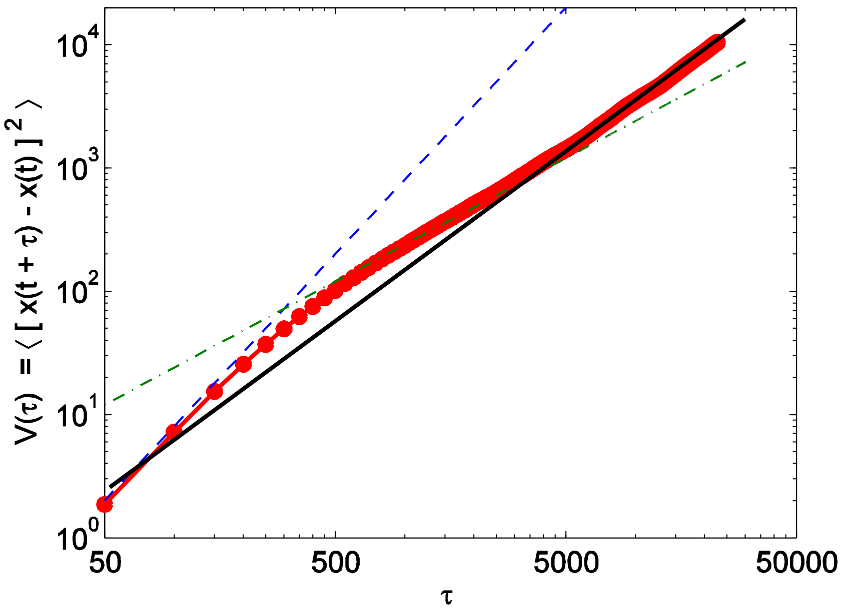
Finite inertia effects are neglected; $\vec{v}$ is compressible due to the spatial dependence of $B(x)$

Garcia *et al* EPS 2005

Particle dynamics

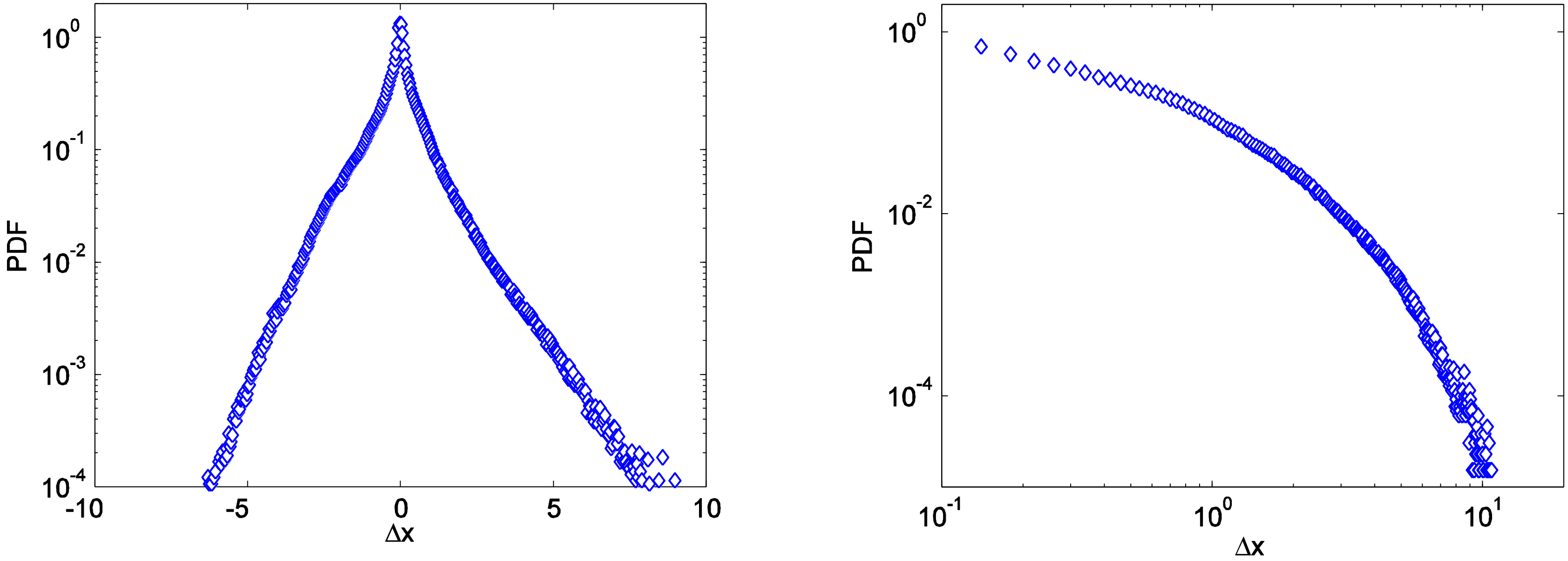


Trajectory of a test particle
released inside LCFS



Variogram of the particle motion,
- - - τ^2; — — — τ; ——— $\tau^{1.4}$

Step size PDF



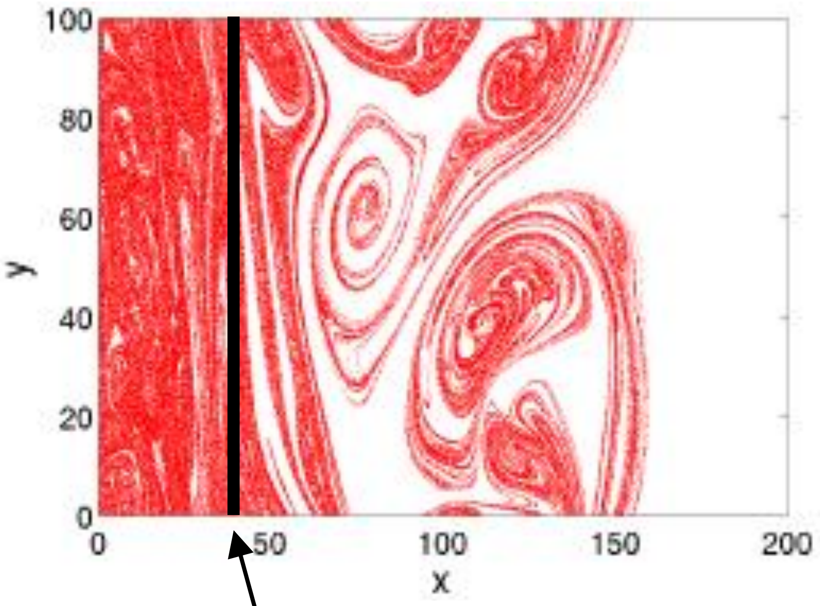
PDF of the radial displacement, Δx, over Δ t = 50; all particles. ‹Δx› = -0.08, standard deviation, σ = 1.02, skewness, S = 0.4, and kurtosis, K = 10.7.

Broad exponentially decaying tails.

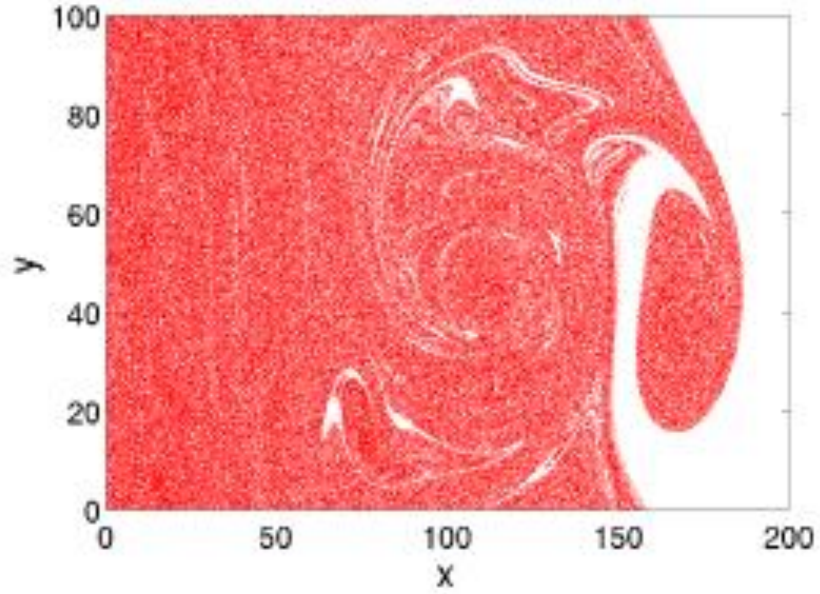
Long steps are almost equally probable in both in- and outgoing directions.

Particle dispersion

t = 10.000

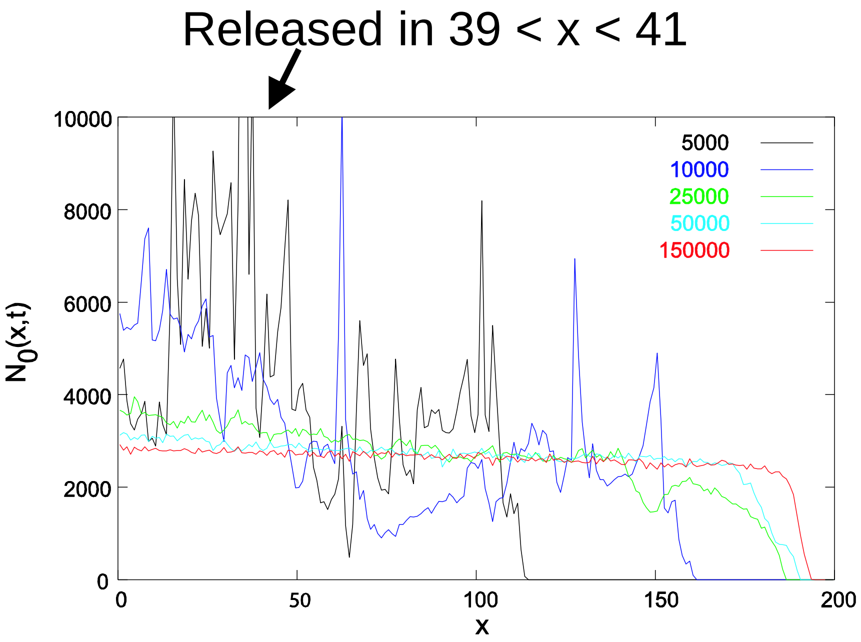


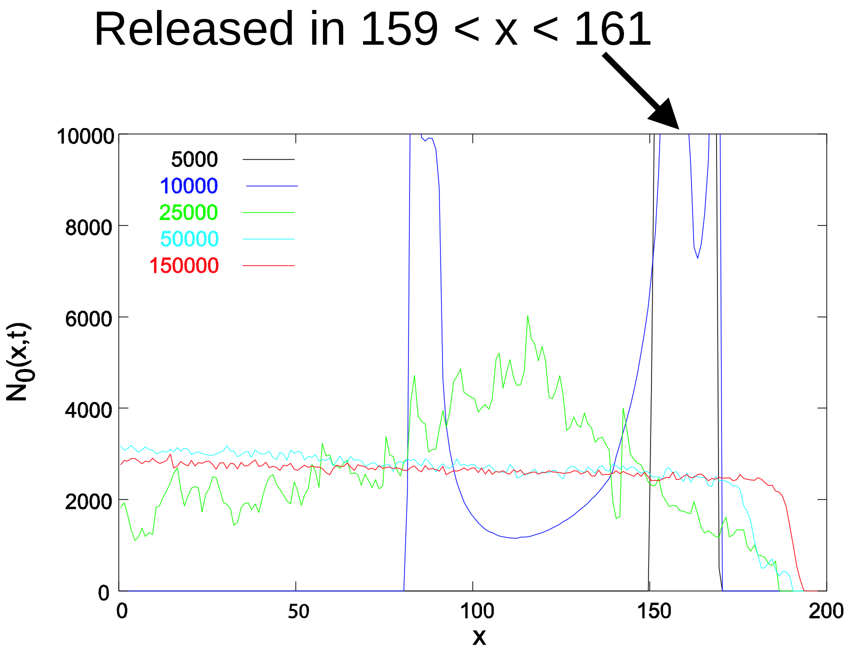
t = 25.000



Particles released at $39 < x < 41$

Evolution of the impurity density





Evolution of the impurity/tracer particle density N_0 averaged over y.

Evolution of the impurity density

Density profile $N_0(x) \propto B(x)$ independent of release position.
The transport is not "Fickian" diffusion. It can be described by an effective pinch:

$$\left(\frac{\partial}{\partial t} + \frac{1}{B} \hat{z} \times \nabla_\phi \cdot \nabla \right) \frac{N}{B} = 0,$$

N/B is a Lagrangian invariant: Effective turbulent mixing: N/B uniformly distributed in space.

Uniform distribution within few burst times

Impurities are effectively mixed by the turbulence in the SOL within a few burst periods. Even if originating far out in the SOL they will quickly penetrate across the LCFS into the edge plasma. corresponding to the so-called inward (curvature) pinch.